Cash Flow Valuation

**Future Value and Compounding**

Future Value – The value that an amount today will be worth at a certain point in the future.

**Simple Interest – The interest earned only on the original amount invested.**

**Compound Interest – Interest earned on the original amount invested plus previously earned interest.**

Understanding the future value of money is a crucial component of sound investment and pricing decisions - including the determination of premiums sufficient to write profitable insurance business. For example, an underwriter receives a premium payment at the beginning of a policy period. The premium amount can be invested and earn a rate of return before the associated losses are paid. Determining the future value of the premium payment is helpful for assessing the adequacy of the premium for paying the losses.

***Question: List the three factors that determine the future value of a monetary sum***

**A monetary sum’s future value depends on its initial or present value (the principal), the applicable rate of return, and the length and number of periods over which it will grow**. If the sum grows over multiple periods, its future value is determined by whether the return is calculated based on the original principal only or the original principal plus the amount earned over previous periods.

**Future Value over a single period**

Suppose an individual wants to know the value of $10,000 deposited in an account that pays 4 percent simple interest annually and left there for one year. The calculation of how much an amount will be worth at the end of a single period uses this equation.

FV1 = PV x (1+r) 10,000 X 1.04 = 10,400

FV= Future Value at the end of a single period

PV = Present value, or value at the beginning of the period

R = Interest rate

Therefore, by investing $10,000 today, the individual earns $400 in interest over the single period (one year), resulting in a future value at the end of the year of $10,400.

An alternate way to calculate future value is to use a future value table to determine the future value factor for a given interest rate. A future value factor is equal to the value of $1 at a given interest rate over a **number of periods (n).** To calculate future value over a single over a single period, this factor can be substituted into the future value formula in this manner.

FV = PV X FVfactor

**Future Value Over Multiple Periods**

When an amount deposited will earn interest for multiple periods, its future value is determined by whether it will earn simple interest or compound interest. As compound interest is earned on an ever-increasing amount, it producers a higher value over multiple periods than will simple interest.

Returning to the example of the future value of $10,000 over a single period, if the principal is left in the account for several years and earns 4 percent simple interest paid annually, the account balance will increase by $400 for each year it remains in the account.

If instead, the bank pays compound interest annually rather than simple interest annual, the interest earned each year will be added to the prior year balance and will earn interest during the next year. One way to calculate the future value is to repeat the calculation of a future value over a single period, each time substituting the revised balance. For example the initial deposit of $10,000 increases to $10,400 then apply the FV calculation to 10,400 and it is $10,816 (10,400 x 1.04)

FVn = PV x (1 + r)

Where FVn = future value at the end of n periods

n = number of periods

Using the formula for the future value at the end of n periods, the future value of the $10,000 principal at 4 percent interest at the end of 5 years can be calculated

FVn = PV X (1+r)n

10,000 X 1.2167 (the value in the table under 4 percent interest rate at the fifth period row)

$12,167 (rounded)

The calculation assumes the interest is payable annually at the end of the year. However, although interest payments can be made other than annually – for example, quarterly at the end of each quarter or semiannually at the end of each six-month period. This means there is more than one payment period each year.

**Question: Why is the number of payment periods important in the calculation of compound interest**

**The number of payment periods is important because the interest earned will be compounded for each period. Assume that the account used in the previous example has an annual interest rate of 4 percent, but that interest is paid semiannually. In this case, the interest on the money is paid every six months rather than once a year. This means, in effect, that the account earns 2 percent every 6 months**. In this situation, to determine how much money there will be at the end of n years, a slightly modified version of the future value equation is used.

FVn = PV X [1 + (r ÷ m) n x m m = number of times per year the interest is paid

With semiannual interest, the future value of the $10,000 principal after one year will be $10,404 rounded to the nearest dollar

FVn = PV X [1 + (r ÷ m) n x m

= 10,000 X [ 1 + (0.04 ÷ 2) (you will then be using the 2% interest rate figure from the table and 2 payment periods)

= 10,000 x (1.02)2 2 = number of periods

= 10,000 x 1.0404 The value at 2 percent interest and 2 periods

= 10,404

At the end of five years, the account will grow to $12,190 (rounded to the nearest dollar)

FVn = PV x [1 + (r ÷ m)] nxm

$10,000 X [1 = (0.04 ÷ 2)] 5x2 5x2 = 10 5 years times number of interest payments per year, so you will be using the value at 10 periods from the table

10,000 x 1.219

$12,190

**The value for this example can also be obtained from the future value table; however, the interest rate used should be r ÷ m (interest rate ÷ number of payments periods) semi-annual = 2, quarterly would be 4, and the number of periods use should be n x m (number of payment periods x number of times per year the interest is paid). This means that the future value factor is 1.2190 which is the factor for an interest rate of 2 percent (0.04÷ 2 = 2%) over ten compounding periods (5 x 2 = 10). Need to remember to change the payment periods as well as the percentage. Semi-annual interest 0.04 becomes 2% and then use the number of payment periods on the table for your factor.**



***Question: Identify two methods insurance professionals may use to determine future values in addition to future value tables***

**As an alternative to using the future value table, insurance professionals often use financial calculators and/or computer spreadsheet programs to determine future values.**

**Effective Annual Interest Rate**

Stated Interest Rate – The quoted annual rate of interest that does not take account of the frequency of compounding.

Effective Annual Interest Rate – The rate of interest that reflects the effect of compounding more than once a year.

When compound interest is paid over a specific period, interest earned during the period is added to the balance at the end of the previous period. This new balance earns interest during the subsequent period, after which the cycle repeats. Therefore; the more frequently compounded interest is paid (for example, annually, semi-annually, or quarterly), the greater the effective annual interest rate.

**Compound interest produces a higher return than simple interest because after the first payment period, compound interest is calculated using a higher principal amount. Compound interest paid more than once a year produces a higher return than compound interest paid annually because interest is earned more often.** The more frequent interest is earned, the more quickly the principal on which the interest is calculated increases.

***Question: Contrast the stated interest rate with the effective annual interest rate***

**By applying compound interest more frequently than annually, the bank is effectively applying a higher annual interest rate than the stated interest rate. The effective annual interest rate (EAR) increases as the number of compounding periods increases because the interest is earned more frequently**.



**Present Value and Discounting**

Present value – the value today of money that will be received in the future

Discounting – The process of calculating the present value of a future amount

Discount rate – the rate of return used in determining the present value of a future sum

Various situations exist in which a risk management or an insurance professional should know the current, or present value of money that will be received or paid out in the future. He or she might have to determine, for example, how much needs to be invested today to generate funds sufficient to pay for loss control equipment at a specific time in the future.

***Question: Explain why money received in the future must be a higher amount than money received presently to be of the same value***

**Because a sum of money grows over time by earning a return (for example, interest on a bond investment), its present value is less than its future value.** The present value of a sum of money to be received in the future depends on the rate of return it could earn if it were received today and the number of periods over which it would earn the rate of return.

**Present Value over a single Period**

The process of calculating present value is called discounting. A discount rate is used to calculate the present value of a future amount. The formula

PV = FVn ÷ (1 + r) Where **n = number of periods and r = rate of return**

**For example, assume that at the end of one year and organization needs $10,300 to be in a savings account that pays 3 percent interest compounded annually.**

**10,300 ÷ (1 + 0.03)1 period**

**10,300 ÷ 1.03 = 10,000**

**Present Value Over Multiple Periods**

**Discounting can also be used to determine the present value of money that will be received or paid out many years in the future. For example, assume an organization has a receivable totaling $11,910 thats due three years from today.**

The financial manager must choose between receiving full payment of the $11,910 in three years or accepting a reduced payment today in full settlement of the receivable. Assuming the money, if received today could be deposited in an account that earns 3 percent interest for three years, **what is the smallest amount the financial manager can accept today to make the second option an acceptable alternative.**  The decision can be made by calculating the present value or discounted value of $11,910

Discounted Value: PV = FVn ÷ (1 + r) n

$11,910 ÷ 1.0927 **Present value divides the payment amount due in the future by the rate found in the (future values at the end of n periods**)

$10,899 is the present value

The present value of $11,910 received three years from now at a discount rate of 3% is $10,899. Therefore, the financial manager views receiving $10,899 today as equally beneficial as receiving $11,910 in three years. If the settlement offer to be received today is less than $10,899, the promise of $11,910 in three years is preferable. Conversely, is the settlement offer is more than #10,899, then settlement is preferable to the promise of $11,910 in three years.

**Another method of determining present value is by using financial tables. The present value factors**

[1 ÷ (1 + r)n] can be obtained from the present value table.

**To determine the present value in the previous example by using the present value table, multiply the future ure value $11,910 by the factor in the (present values percent interest rate column at the third period row.**

**11,910 X 0.09151 = $10,899 (rounded)**

**An alternative to using the present value table, risk management and insurance professionals often use financial calculators and/or computer spreadsheet programs to determine present values**.

**Rate of Return on an Investment**

**The rate at which an investment amount will grow, and the length of time over which it will grow, determine its future value.** Therefore, it is important to be able to calculate the rate of return necessary for an amount to accumulate to a required future value.

**The relationship between an invested sum’s Present Value (PV), Future Value (FV), and the expected Rate of Return ® on the investment may be expresses as PV = FVn** **÷ (1=r)n.** Assume an investment of $15,000 today is expected to mature in ten years with a value of $29,505. What is the annual rate of return ® that will be earned on this investment?

PV = FVn ÷ (1 + r) n

$15,000 = $29,505 ÷ (1 + r)10

(1+r)10 = $29,505 ÷ $15,000

(1 + r)10 = 1.967

**The discount rate may then be determined by using a future value table**. To calculate, take the value of 1.967 (which you got by dividing $29,505 amount at maturity by $15,000 the initial investment) and using the future value table find the percentage using the 10 year (or 10 periods) to find your interest rate in the Future Value Table.

**By knowing the future value of a sum invested today, an organization can determine whether the investment meets the organization’s required rate of return. For example, assume the business is considering an investment of $10,000 that will grow to $18,000 in 8 years. The business requires a 7% annual rate of return on its investment. It uses this calculation to determine whether to make the investment.**

**18,000 ÷ 10,000 = 1.8 using the future value table 7%=1.7182 and 8% = 1.8509**

**They are looking at a 7-8% return on investment, meeting the 7% required return**.

**Part 5 - Future and Present Values on an Ordinary Annuity**

**Three common types of annuities are ordinary annuities, annuities due, and perpetuities**. Of these, ordinary annuities are the most prevalent. Therefore, in practice, the term “annuity” is used to refer to an ordinary annuity. **Annuities can also be classified as either immediate or deferred. As the name suggests, immediate annuities begin to make fixed payments immediately, and deferred annuities provide for an accumulation period before the series of payments begins**.

An ordinary annuity is a series of equal periodic payments made or received at the end of each period. The future value of an ordinary annuity is equal to the total of the future values of each of the payments made or received. Therefore, **an ordinary annuity’s future value depends on the fixed periodic payment amount, the number of periods over which the payments are made or received, and the rate of return on the payments**. The periodic return is compounded from one period to the next.

For example, an ordinary annuity may consist of payments of $500 received at the end of each year for four years. Assume that each payment will be deposited in an account that pays 6 % interest annually.

**Using the Future Value of an Annuity Table you will go to the 6% interest rate and 4 year period the rate is 4.3746 X $500 = $2,187.3**

**FVA = A X FVAG**

**Future value of Annuity = Amount of payment x Future value of Annuity interest rate amount**

As a practical example of how the calculation of the future value of an ordinary annuity can be used, assume that a risk management professional has determined that the safety guards on the production machinery of a large manufacturer will need to be replaced in 5 years. The total cost of the replacement at that time is expected to by $50,000

**The risk management professional wants to budget an equal amount to be deposited into a reserve fund at the end of each of the next 5 years. The fund is expected to earn a return of 6% compounded annually. The risk management professional knows the amount to be deposited will be less than $10,000 ($50,000 ÷ 5 years) because of the benefit of the 6% compounded interest.**

**A = FVA ÷ FVAF**

**A = $50,000 ÷ 5.371 (FVAF rate 6% at 5 periods)**

**A = 8,869.808 or $8,870**

**Present Value of an Ordinary Annuity**

**As well as calculating future value, a financial manager often must determine the present value of an ordinary annuity – that is, the value today of a series of equal payments to be made or received in the future at the end of each specified period**.

Suppose a manger is offered payments of $750 per year for three years in satisfaction of a current account receivable of $2,100. Also assume that the $2,100 sum, if received today could be deposited in a savings account that pays 6% interest. Therefore, the opportunity cost, or lost return, of receiving payments in the future rather than today is 6% per annum. What is the equivalent value today of these consecutive payments.

**Using the present value of the annuity table, multiply the payment per period $750 by the table value in the 6% interest column at the third period row. 2.6730 = $2,004.75**

**PVA = A x PVAF**

Because the present value of the annuity is less than the present value of the account receivable, the manager should not accept the offer.

Another example of the usefulness of determining the present value of an annuity, consider the case of an underwriting manager who is planning to retire and must choose between taking an immediate lump sum of $250,000 on an ordinary annuity that pays $35,000 per year for ten years. Assume that 5% is an achievable rate of return based on the retiree’s conservative investment philosophy. Also, assume no income taxes would apply in this calculation.

**To be able to make an informed decision, the future retiree must determine the present value of the annuity to be able to compare it to the lump sum payment. The present value can be solved using the present value of an annuity table. Multiplying the present value of an annuity factor in the 5% column in the 10th period row (7.7217 by the $35,000 payment expected yearly for those 10 years.**

**35,000 x 7.7217 = $270,260**

**The present value of $270,260 is greater than the lump sum payment of $250,000. Therefore, based on these assumptions, the retiree should accept the annuity rather than the lump sum**.

**Part 6 - Annuities Due**

The primary distinction between an ordinary annuity and an annuity due is the timing of their payments. **Payments received or paid out under an ordinary annuity occur at the end of each period, while payments received or paid out under an annuity due occur at the beginning of each period**. **Examples of annuities due include lease agreements, lottery payouts, and retirement plan payouts.**

**Future Values of an Ordinary Annuity and Annuity Due**

**Calculating the future or present value of an annuity due can be done by totaling the future value or present value of each of the payments**. This method can also be used to value ordinary annuities. For **the future value of an ordinary annuity, the total time over which interest is compounded for each period is less than the time under an annuity due**. This is because the payments occur at the end rather than at the beginning of each period.

In the annuity due, the first payment is received immediately, and the subsequent 3 payments are each received at the beginning of each payment period. An annuity due has one more earning period for each of the payments than an ordinary annuity due. Therefore, all other assumptions being equal, the future value of an annuity will always be higher than an ordinary annuity’s. A simple way to calculate the future value of an annuity due is to determine its future value, assuming it was an ordinary annuity, and then multiply the result by (1 + r) where r is the annuity’s rate of return, to account for the additional earning period.

FV(annuity due) = FV(ordinary annuity x (1 + r)

**Present Values of an Ordinary Annuity and an Annuity Due**

The difference in timing of payments also affects the present value calculations for an ordinary annuity and an annuity due. Just as the future value of an annuity due is greater than the future value of an ordinary annuity, the present value of an annuity due is greater than the present value of an ordinary annuity.

With payments at the beginning of each period rather than at the end, less discounting to present value is required with an annuity due than with an ordinary annuity.

**The alternate method of determining the future value of an annuity due can also be used to determine the present value of an annuity due: determine the present value (assuming it is an ordinary annuity), and multiply the result by (1 + r) to account for the additional earning period**.

PV Annuity Due = PV ordinary annuity x (1 + r)

PV = $2,004.76 x (1 + 0.06)

PV = 2,004.76 x 1.06

PV Annuity Due = 2,125.05

**Part 7 - Perpetuities**

Some annuities do not have a date on which payments stop; payments continue indefinitely. Annuities that go on forever in this manner are called perpetuities.

**The dividends paid on a preferred stock are an example of a perpetuity. Preferred stock normally does not have a maturity date at which time the investment will be repaid to the owner; it life is indefinite.** Because a company must pay the dividends on preferred stock before any common stock dividends can be paid, it is likely that the preferred dividend will be paid every year into the future.

**To determine whether it is preferable to receive a preferred stock dividend or to make an alternative investment requires calculating the perpetuity’s present value. This is the formula for the present value of a perpetuity:**

**PVP = A ÷ r where: A = Payment Amount (dividend) and r = discount rate**

For example, if a preferred stock pays an annual dividend of $6, and 5% per year can be earned on alternative investments with similar risks, what is the present value of the preferred stock

PVP + A **÷** r **or $6.00 ÷ 0.05 = $120**

**Another scenario in which the calculation of the present value of a perpetuity is used is when valuing income-producing real estate, such as rental properties. This type of valuation assumes that the net operating income (income before financing costs) from the rental property will continue in perpetuity.**

**The discount rate used to fine the present value of the net operating income is the capitalization rate, commonly called the cap rate. An individual’s cap rate may be his or her required rate of return on an investment. However, a market cap rate may also be calculated by dividing the income streams of similar properties by the value at which those properties have recently been sold**.

As an example, assume a rental property generates annual net operating income of $100,000 and that the financial manager of that property requires a 9% rate of return.

**The present value of the property is calculated in this manner:**

PVP = A **÷** r $**100,000 ÷ 0.09 which = $1,111,111**

**Part 8 - Present Value of Unequal Payments**

Not all investment decisions involve streams of payments that are equal. In some, payments that will be made or received vary substantially from period to period.

The valuation methods required to calculate the present value of a stream of unequal payments differs from those used to calculate the present value for a stream of equal payments (an annuity**). With unequal payments, the present value of each individual payment must be calculated and the results summed.**  For example assume an individual is offered $250 one year from now, $300 two years from now, $500 three years from now and $700 4 years from now. The present value of this future stream of payments can be calculated using the present value table.

Assuming a 6% rate of return, the exhibit shows how the present value of this stream of unequal payments can be calculated. **The present value of each individual payment is calculated by the corresponding present value factor from the present value table and then summing the individual present values.**

1st payment $250 X (6% rate period 1) 0.9434 = $235.85

2nd payment $300 X (6% rate period 2) 0.890 = $267.00

3rd payment $500 X (6% rate period 3) 0.8396 = $419.80

4th payment $700 X (6% rate period 4) 0.7921 = $554.47

Summed Present value of unequal payments $1,477.12

**Part 9 - Net Present Value**

The net present value technique for evaluating an investment is based on discounting the related cash outflows and inflows using a required rate of return. For example, a risk management professional may want to determine whether investing today in a machinery maintenance program (a cash outflow) will save costs from machinery breakdown incidents in the future (a form of cash outflow based on saving expenses).

**An investment’s net present value (NPV) is the difference between the present value of its cash inflows and the present value of its cash outflows**. The rate of return used for discounting ® is the rate of return required by the investor. When making investment choices, organizations generally adhere to what is known as **the NPV rule, which dictates that an investment should be made only if the NPV is greater than zero (0).** **The organization investing in a project usually sets its required rate of return equal to its cost of capital so as to only accept investments that cover its cost of funds**.

The NPV calculation is represented as an equation:

NPV = Co + (Ct ÷ (1 = r) + (Cn ÷ (1 + r),

Where Co = Cash flow at beginning of project,

Ct = Payment at period t for t = 1, through t = n,

r = discount rate,

n = number of periods

Note the cash outflow is a negative number and the cash inflows are positive. Furthermore, the equation assumes there is a single cash outflow (investment) made today. The equation would differ if the investment called for a series of cash outflows (investments) to be made at various point in time.

For example, assume that a risk management professional is determining whether to invest $10,000 today in a 3 year machinery maintenance project. The company requires a rate of return of 7%. Further assume that it expects to save breakdown expenses of $2,500 at the end of the first year, $3,300 at the end of the second year, and $4,700 at the end of the third year.

The present value table may be used in the determination of the NPV.

**The present value of each payment may be calculated by multiplying the payment amount by the appropriate present value factor. The present value factor lies at the intersection of the corresponding period and interest rate.**

Year 0 $10,000 1.000 value factor -$10,000

Year 1 $2,500 0.9346 $2,336

Year 2 $3,300 0.8734 $2,882

Year 3 $4,700 0.8163 $3,837

Net Present Value - $945

Because the investment’s NPV is negative, the NPV rule suggest that the organization should not make this investment because the present value of its cash inflows does not exceed the present value of its cash outflows. If the proposed investment’s NPV were positive, the NPV rule would dictate that the investment should be made.

NPV analysis and the NPV rule should not be used as the sole determinant as to whether to make an investment. In the machinery maintenance example, the risk management professional may have nonfinancial reasons for avoiding machinery breakdowns. Alternatively, perhaps not all the cost related to machinery breakdowns were factored into the NPV analysis**. Some of the limitations of NPV analysis that should be considered are these:**

* **The amounts and timing of cash flows may differ from those expected over the life of the investment**
* **NPV analysis does not formally factor in the effect of uncertainty (risk) with respect to future cash flows, losses, discount rates, or time horizons.**
* **NPV analysis focuses on maximizing economic value and disregards an organization’s nonfinancial goals and other stakeholder’s interest.**